NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF AGRICULTURE & NATURAL RESOURCES SCIENCES

QUALIFICATION : BACHELOR OF SCIENCE IN AGRICULTURE	
QUALIFICATION CODE: 07BASA	**LEVEL:** 6
COURSE CODE: MTA611S	**COURSE NAME:** Mathematics for Agribusiness
DATE: June 2022	**PAPER:** Theory
DURATION: 3 Hours	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	Mr. Mwala Lubinda
MODERATOR:	Mr. Teofilus Shiimi

INSTRUCTIONS
1. Attempt all questions 2. Write clearly and neatly. 3. Number the answers clearly & correctly.

PERMISSIBLE MATERIALS

1. All written work MUST be done in blue or black ink
2. Calculators allowed
3. No books, notes and other additional aids are allowed

THIS QUESTION PAPER CONSISTS OF 7 PAGES (<u>including</u> this front page).

QUESTION ONE	[MARKS]
a. Give concise definitions of the following concepts related to functions:	
i. Range	(2)
ii. Domain	(2)
b. Let $f(a) = (a^2 - 2a + 6)^{\frac{1}{2}}$, compute $f(1)$ and $f(-1)$.	(2)
c. Use interval notation to express the domain and range of the following function: $$g(k) = \frac{2k - 1}{k^2 - k}$$	(6)
d. Suppose you know that the production function that expresses the relationship between table grapes output (q) and fertilizer application rate (x) is a quadratic function that has: (i) maxima point and (ii) roots at 0 and 75. Based on the provided information, answer the questions below	
i. Derive the mathematical equation of the production function.	(3)
ii. Find the critical point of the production function you have derived in d(i).	(5)
iii. Draw and label a graph that illustrates the production function. The graph must clearly show the roots, maxima, and y-intercept points of the production function.	(5)
TOTAL MARKS	**[25]**

QUESTION TWO	[MARKS]
a. Use mathematical expressions to concisely define the following concepts:	
i. Newton's Difference Quotient.	(2)
ii. Regular limit.	(2)
b. Briefly describe at least two algebraic approaches you would use to find the limit of function at a given point, $x = a$.	(4)
c. Find:	
i. $\lim_{h \to 0} \frac{(2+h)^2 - 4}{h}$	(2)
ii. $\lim_{L \to 1} \sqrt{\frac{L-1}{L^2+2L-3}}$	(3)
iii. $\lim_{x \to 6} \frac{\sqrt{x-2} - 2}{x-6}$	(5)
d. Find the equation of a straight-line that is tangent to the curve: $$y = q^2 - 2q - 24$$ at $q = 4$.	(7)
TOTAL MARKS	**[25]**

QUESTION THREE	[MARKS]

a. Define the following concepts:

   i. Partial derivative (2)

   ii. Cross derivative (2)

b. Find the first derivative of the following function:

   i. $f(x) = (3x^4 - 5)^6$ (3)

   ii. $f(L) = \sqrt[3]{\frac{L-1}{L^2+2L-3}}$ (4)

c. Given a function:

$$z(x, y) = 3e^{7-2x}y^2$$ (6)

   Find z_x, z_y and z_{yz}.

d. Optimize the following function by (i) finding the critical value(s) at which the function is optimized and (ii) testing the second-order condition to distinguish between a relative maximum or minimum. (8)

$$q(x) = x^3 - 6x^2 - 135x + 4$$

TOTAL MARKS	[25]

QUESTION FOUR	[MARKS]

a. Find:

i. $\int \frac{1}{\sqrt[3]{t}} dt$ (2)

ii. $\int_0^1 (3x^3 - x + 1)\, dx$ (3)

iii. $\int 12x^2(x^3 + 2)dx$ (5)

b. In the manufacture of a product, fixed costs N$4000. If the marginal-cost function is:

$$\frac{dc}{dq} = 250 + 30q - 9q^2$$ (5)

where c is the total cost (in dollars) of producing q kilograms of product. Find the cost of producing 10 kilograms of the product.

c. To fill an order for 100 units of its product, a firm wishes to distribute production between its two plants, plant 1 and plant 2. The total-cost function is given by:

$$c = f(q_1, q_2) = q_1^2 + 3q_1 + 25q_2 + 1000$$

where q_1 and q_2 are the numbers of units produced at plants 1 and 2, respectively. How should the output be distributed to minimize costs? *(Hint: assume that the critical point obtained corresponds to the minimum cost and the constraint is $q_1 + q_2 = 100$).* (10)

TOTAL MARKS	[25]

THE END

FORMULA

Basic Derivative Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Derivative Rules for Exponential Functions

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(e^{g(x)}) = e^{g(x)}g'(x)$

$\frac{d}{dx}(a^{g(x)}) = \ln(a)\, a^{g(x)}\, g'(x)$

Derivative Rules for Logarithmic Functions

$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$

$\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}$

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}, x > 0$

$\frac{d}{dx}(\log_a g(x)) = \frac{g'(x)}{g(x) \ln a}$

Basic Integration Rules

1. $\int a\, dx = ax + C$
2. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{x} dx = \ln|x| + C$
4. $\int e^x\, dx = e^x + C$
5. $\int a^x dx = \frac{a^x}{\ln a} + C$
6. $\int \ln x\, dx = x \ln x - x + C$

Integration by Substitution

The following are the 5 steps for using the integration by substitution metthod:

- Step 1: Choose a new variable ***u***
- Step 2: Determine the value ***dx***
- Step 3: Make the substitution
- Step 4: Integrate resulting integral
- Step 5: Return to the initial variable ***x***

Integration by Parts

The formula for the method of integration by parts is:

$$\int u dv = u \cdot v - \int v du$$

There are three steps how to use this formula:

- Step 1: identify ***u*** and ***dv***
- Step 2: compute ***u*** and ***du***
- Step 3: Use the integration by parts formula

Unconstrained optimization: Univariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

- Step 1: Find the critical value(s), such that:

$$f'(a) = 0$$

- Step 2: Evaluate for relative maxima or minima
    - If $f''(a) > 0 \rightarrow$ minima
    - If $f''(a) > 0 \rightarrow$ maxima

Unconstrained optimization: Multivariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

Relative maximum	*Relative minimum*
1. $f_x, f_y = 0$	1. $f_x, f_y = 0$
2. $f_{xx}, f_{yy} < 0$	2. $f_{xx}, f_{yy} > 0$
3. $f_{xx} \cdot f_{yy} > (f_{xy})^2$	3. $f_{xx} \cdot f_{yy} > (f_{xy})^2$

Constrained Optimization

The following are the steps for finding a solution to a constrained optimization problem using the Langrage technique:

- Step 1: Set up the Langrage equation
- Step 2: Derive the First Order Equations
- Step 3: Solve the First Order Equations
- Step 4: Estimate the Langrage Multiplier

Additionally:

- If $f_{xx} \cdot f_{yy} < (f_{xy})^2$, when f_{xx} and f_{yy} have the same signs, the function is at an **inflection point**; when f_{xx} and f_{yy} have different signs, the function is at a **saddle point.**
- If $f_{xx} \cdot f_{yy} = (f_{xy})^2$, the test is **inconclusive.**